# CHAPTER 3. SNOW ACCUMULATION AND ABLATION MODEL

## 3.1 INTRODUCTION

This chapter describes the basic model representing the physical processes which are needed to simulate the accumulation and ablation of a snowpack. The basic philosophy of the model is that each significant physical component be represented separately, rather than to use a single index to explain several processes, e.g., the use of degree-day-factors as described by Linsley *et al*. (1958). As noted in Chapter 1, air temperature and precipitation are the only meteorological variables that are required for this model. Guidelines for determining model parameters are not included in this chapter. Model parameter guidelines are included in the discussion of model calibration in Chapter 5.

## 3.2 FLOWCHART

Figure 3-1 shows a flowchart of the snow accumulation and ablation model. This flowchart shows each of the physical components which are represented in the model. These include, accumulation of the snowpack, heat exchange at the air-snow interface, areal extent of snow cover, heat storage within the snowpack, liquid-water retention and transmission, and heat exchange at the soil-snow interface.

## 3.3 DESCRIPTION OF MODEL COMPONENTS

This section describes the mathematical relationships which are used to model each of the basic components of the snow accumulation and ablation process. It should be noted that in the model all snowpack variables are expressed in terms of mean values over the entire area. Thus, if the total snowpack water equivalent is computed as 6.30 inches and the areal extent of snow cover is 50 percent, then the mean water equivalent over the area actually covered by snow is 12.60 inches.

### 3.3.1 ACCUMULATION OF THE SNOWPACK

The first decision which must be made is whether precipitation entering the model is in the form of rain or snow. Air temperature is used as the index to the form of precipitation. The parameter PXTEMP is the delineation point between rain and snow.

$$TA > PXTEMP \text{ Precipitation is rain, and}$$
$$TA \leq PXTEMP \text{ Precipitation is snow,}$$

where:

TA is the air temperature in degrees F, and
PXTEMP is in degrees F.

For heat storage computations or for computing the melt caused by rain water, the temperature of the precipitation is assumed to be equal to the air temperature. When snow is falling at air temperatures greater than 32°F, the temperature of the snow is set to 32°F.

In order to simulate the accumulation of the snowpack correctly, not only does the form of precipitation need to be determined, but the amount of precipitation must be reasonably accurate. The catch of a precipitation gage can be in error by a considerable amount during snowfall events; especially if the gage is not shielded or if the gage is exposed to high winds. The parameter SCF is used to correct for gage catch deficiency during snowfall, i.e.,

$$PX_a = SCF \cdot PX_g, \tag{3-1}$$

where:

$PX_g$ is the precipitation as recorded by the gage in inches, and
$PX_a$ is the actual water equivalent of the snowfall in inches.

In this model SCF is a mean gage catch deficiency correction factor. For an individual storm $PX_a$ can be in error because of variations in wind speed and direction. However, as the number of storms contributing to the snowpack becomes large, the errors from individual storms will tend to cancel each other.

### 3.3.2 HEAT EXCHANGE AT THE AIR-SNOW INTERFACE

The heat exchange at the air-snow interface is the most critical factor controlling the ablation of a snowpack. This model uses air temperature as the index to the heat exchange mechanisms which control heat flow into or out of the snowpack. There are two basic situations for which heat exchange needs to be estimated: (1) when the air is warm enough so that melt takes place at the snow surface, and (2) when the air is too cold for melt to occur.

#### 3.3.2.1 Melt at Snow Surface

The model assumes melt can occur at the snow surface when the air temperature is above 32°F. The relative importance of various heat exchange mechanisms varies with meteorological conditions. Since only air temperature and precipitation are assumed known in this model, it is impossible to distinguish each condition. However, the rate of melt during rain can be separated from the rate of melt during other conditions. In the model the equation for melt during rain is used when the amount of rain exceeds 0.1 inch in six hours.

a. Melt during rain. During rain several assumptions are made so that melt can be computed from an energy balance equation. The assumptions are: (1) solar radiation is zero, (2) incoming longwave radiation equals the blackbody radiation at the ambient air temperature, (3) snow surface temperature is 32°F, (4) dew point is equal to ambient air temperature, and (5) temperature of the rain water is equal to the ambient air temperature.

A brief derivation of the energy balance equation is as follows:

1. The energy balance of a melting snowpack can be expressed as:

$$M = Q_n + Q_e + Q_h + Q_{P_X} , \qquad (3.1)$$

where: $Q_n$ = net radiative heat transfer,
$Q_e$ = latent heat transfer,
$Q_h$ = sensible heat transfer,
$Q_{P_X}$ = heat transfer by rain water, and
$M$ = amount of melt.

Units of all quantities are inches water equivalent.

2. Based on the preceding assumptions, net radiative transfer during rain on a melting snowpack is:

$$Q_n = \sigma \cdot T_{aK}^4 - \sigma \cdot T_{sK}^4 , \qquad (3.2)$$

where: $\sigma$ = Stefan Boltzmann constant ($5.78 \times 10^{-10}$ inches of melt $\cdot$ day$^{-1}$ $\cdot$ $^{o}K^{-4}$),
$T_{aK}$ = ambient air temperature $^{o}K$, and
$T_{sK}$ = snow surface temperature $^{o}K$ (in this case $T_{sK} = 273^{o}K$).

Assuming linearity of $\sigma \cdot T_{aK}^4$ over the temperature region of main interest, Eq. 3.2 can be expressed as:

$$Q_n = 0.007 \cdot (T_a - 32) , \qquad (3.3)$$

where: $T_a$ is the ambient air temperature, $^{o}F$, and $Q_n$ is in terms of inches/6 hr.

Eq. 3.3 yields values within 5 percent of Eq. 3.2 over the temperature range $32^{o} < T_a < 75^{o}F$.

3. A Dalton-type equation is commonly used to compute vapor transfer. During a rain on snow event, condensation will occur, thus the equation for vapor transfer would be:

$$V = f(u) \cdot (e_a - e_s), \qquad (3.4)$$

where: $V$ = condensation - inches/6 hr.,
$f(u)$ = wind function - inches/(inches $H_g$ $\cdot$ 6 hr.),

$e_a$ = vapor pressure of air - inches $H_g$, and
$e_s$ = vapor pressure of snow surface - inches $H_g$ (assumed to be the saturation vapor pressure at the snow surface temperature = 0.18 in. $H_g$ at 32°F).

Thus, the latent heat transfer during a rain on snow event is:

$$Q_e = L_V \cdot V, \tag{3.5}$$

where: $L_V$ = latent heat of vaporization (7.5 inches of melt/inch of condensate).

Combining Eqs. 3.4 and 3.5, the equation for latent heat transfer during a rain on snow event is:

$$Q_e = 7.5 \cdot f(u) \cdot (e_a - 0.18), \tag{3.6}$$

where: $Q_e$ is in inches/6 hr.

However, for every 7.5 inches of latent heat melt, one inch of condensate is also added to the snowpack. Thus, the total amount of liquid water produced by latent heat exchange during a rain on snow event ($W_{Q_e}$) is:

$$W_{Q_e} = 8.5 \cdot f(u) \cdot (e_a - 0.18), \tag{3.7}$$

where: $W_{Q_e}$ is in inches/6 hr.

4. If it is assumed that the eddy transfer coefficients for heat and vapor are equal, then the ratio of $Q_h/Q_e$, commonly referred to as Bowen's ratio, can be expressed as:

$$\frac{Q_h}{Q_e} = \gamma \cdot \frac{T_a - T_s}{e_a - e_s}, \tag{3.8}$$

where: $\gamma$ is the psychrometric constant - inches $H_g$/°F ($\gamma$ = 0.000359·PA where PA is atmospheric pressure - in. $H_g$), and $T_s$ is the snow surface temperature - °F.

Substituting Eq. 3.6 for $Q_e$, the expression for sensible heat transfer becomes:

$$Q_h = 7.5 \cdot \gamma \cdot f(u) \cdot (T_a - 32) \ . \qquad (3.9)$$

5. The heat transferred by rain water to the snow is the difference between the initial and final heat content of the rain water. This can be expressed as:

$$Q_{px} = C_p \cdot P_x \cdot T_a - C_p \cdot P_x \cdot 32^{\circ}F, \qquad (3.10)$$

where: $C_p$ = specific heat of water, 0.007 inches water equivalent/ $^{\circ}F$, and
$P_x$ = amount of precipitation - inches.

$C_p = \frac{\text{Specific heat}}{\text{heat of fusion}}$

units, $^{\circ}F^{-1}$

Thus, the melt caused by rain water is:

$$Q_{px} = 0.007 \cdot P_x \cdot (T_a - 32). \qquad (3.11)$$

$\frac{1 \text{ cal}}{\text{gm } ^{\circ}\text{C}} \cdot \frac{\text{gm}}{80 \text{ cal}} \cdot \frac{^{\circ}\text{C}}{1.8^{\circ}\text{F}} = .007$

Substituting Eq. 3.3, 3.6, 3.9 and 3.11 into Eq. 3.1 and including the amount of condensate, the equation used in the model for melt during a rain on snow event becomes:

$$M = 0.007 \cdot (T_a - 32) + 7.5 \cdot \gamma \cdot UADJ \cdot (T_a - 32),$$
$$+ 8.5 \cdot UADJ \cdot (e_a - 0.18) + 0.007 \cdot P_x \cdot (T_a - 32), \qquad (3.12)$$

where: UADJ is a parameter representing the average six-hour wind function during rain on snow events, and M is in units of inches/6 hr.

b. Melt during non-rain periods. During non-rain periods melt at the snow surface is assumed to be linearily related to the difference between the air temperature and a base temperature, MBASE (units are $^{\circ}F$). The most commonly used base temperature is $32^{\circ}F$. Thus, melt during non-rain periods can be expressed as:

$$M = M_f \cdot (T_a - MBASE), \qquad (3.13)$$

where: $M_f$ = melt factor - inches/(6 hr. $\cdot$ $^{\circ}F$).

This relationship is adequate for any single period of the snow season. However, the melt factor for one portion of the snow season differs from the melt factor for other portions because of the changing relationship between the meteorological factors which affect melt and the quantity ($T_a$ - MBASE). Thus, the model uses a seasonally varying melt-factor. The minimum melt factor (MFMIN) is assumed to occur on December 21 and the maximum melt factor (MFMAX) on June 21. A sine curve is used to extrapolate melt factors for other dates, as shown in Figure 3-2.

### 3.3.2.2 Heat Exchange During Non-Melt Periods

When the air temperature is below 32°F the model assumes melt does not occur. In this situation the heat exchange can be positive (snowpack gaining heat) or negative (snowpack losing heat). The direction of heat flow depends on whether the air is warmer or colder than the surface layer of the snowpack. An antecedent temperature index (ATI) is used as an index to the temperature of the surface layer of the snowpack. This index is computed as follows:

$$ATI_2 = ATI_1 + TIPM \cdot (T_{a_2} - ATI_1), \qquad (3.14)$$

where: subscripts refer to time period one and two. TIPM is an anteceden temperature index parameter ($0.0 < TIPM \leq 1.0$).

Exceptions to Eq. 3.14 are:

a. When ATI is greater than 32°F, ATI is set to 32°F.
b. When the snowpack is isothermal at 32°F, ATI is set to 32°F.
c. When more than 0.2 inches water equivalent of snowfall occurs in six hours then ATI is set equal to the temperature of the new snow, since the new snow is now the surface layer.

The heat exchange during a non-melt period is assumed proportional to the temperature gradient defined by air temperature and the antecedent temperature index. Thus the change in the heat storage of the snowpack when $T_a < 32°F$ is:

$$\Delta HS_2 \doteq NM_f \cdot (T_{a_2} - ATI_1), \qquad (3.15)$$

where: $\Delta HS$ = change in snowpack heat storage - inches water equivalent/ 6 hr., and
$NM_f$ = negative melt factor - inches/(6 hr. · °F).

Subscripts refer to time periods and indicate that $\Delta HS$ is calculated using the value of ATI at the end of the previous six-hour period.

The conduction of heat into or out of the snowpack is primarily a function of snow density in addition to the temperature gradient. The density of the upper layer of the snowpack is variable, but tends to increase as the snow "ripens" and melt progresses. Thus, the negative melt factor should vary seasonally. Since heat transfer during non-melt periods is much less significant than during melt periods, additional mathematical relationships and parameters to describe this seasonal variation are not warranted. In this model the same seasonal variation used for the non-rain melt factor is used for the negative melt factor. Therefore, the only parameter needed for non-melt heat exchange is NMF, the maximum negative melt factor. The minimum negative melt factor ($NMF_{min}$) is:

$$NMF_{min} = NMF \cdot \frac{MFMIN}{MFMAX} \tag{3.16}$$

and the seasonal variation is the same as for the non-rain melt factor, as shown in Figure 3-2.

To conclude this section, Table 3-1 summarizes the calculation of heat exchange at the air-snow interface for each heat exchange situation.

### 3.3.3 AREAL EXTENT OF SNOW COVER

The percent of the area which is covered by snow must be estimated to determine the area over which heat exchange is taking place and, in the case of rain on snow, to determine how much rain falls on bare ground. The areal depletion of snow is predominantly a function of how much of the original water-equivalent of the snowpack remains. Because of a similarity in accumulation versus elevation and vegetal cover and a similarity in drift patterns from year to year, each area should have a reasonably unique areal depletion curve. An areal depletion curve, as used in the model, is a plot of the areal extent of snow cover versus the ratio of mean areal water equivalent to an index value, $A_i$ (units are inches water equivalent). The index value, $A_i$, is the smaller of: 1) the maximum water equivalent since snow began to accumulate, or 2) a preset maximum (SI). SI is thus the mean areal water equivalent above which there is always 100 percent snow cover. A typical areal depletion curve is shown in Figure 3-3.

The one problem that remains is the case when new snow occurs over an area that is partially bare. In this case, the area reverts to 100 percent cover for a period of time, then returns to the point where it was on the areal depletion curve before the snowfall occurred. The method of modeling this situation also is shown on Figure 3-3. The variables are defined as follows:

SBAESC = the areal extent of snow cover from the areal depletion curve just prior to the new snowfall;
SB = the areal water equivalent just prior to the new snowfall;
S = the amount of the new snowfall - inches water equivalent; and
SBWS = the amount of water equivalent above which 100 percent areal snow cover temporarily exists.

SBWS is computed as:

$$SBWS = SB + 0.75 \cdot S. \tag{3.17}$$

Thus, the areal extent of snow cover remains at 100 percent until 25 percent of the new snow melts. In reality this 25 percent figure varies from area to area, but the magnitude of the variation and the effect on model results do not warrant the inclusion of another parameter.

### 3.3.4 SNOWPACK HEAT STORAGE

The model keeps a continuous accounting (on a six-hour basis) of the heat storage of the snowpack. The upper limit for heat storage computations is 32°F. Thus, when the snowpack is isothermal at 32°F, the snowpack heat storage is assumed to be zero. When heat is transferred from the snow to the air, heat storage becomes negative. This is called negative heat storage (NEGHS) in the model. Enough heat must be added to bring negative heat storage back to zero before surface melt water or rain water can contribute to liquid water storage or snowpack outflow. Negative heat storage can physically consist of snow at a temperature less than 32°F or refrozen liquid water or a combination of these. It makes no difference what the physical form of negative heat storage is, it is the total amount of the heat deficit that is important.

### 3.3.5 LIQUID-WATER RETENTION AND TRANSMISSION

Snow crystals retain liquid-water similar to soil particles. In the model the maximum amount of liquid-water (LIQWMX - inches) that the snowpack can hold is:

$$\mathrm{LIQWMX} = \mathrm{PLWHC} \cdot \mathrm{WE}, \tag{3.18}$$

where: PLWHC = percent (decimal) liquid-water holding capacity; and
WE = water equivalent of the solid portion of the snowpack in inches.

The model assumes PLWHC is a constant for all snowpack conditions, since variations of liquid-water holding capacity with regard to density and crystal structure are not well defined. The amount of liquid-water that exists within the snowpack at any time is LIQW (units are also inches).

Equations for the transmission of excess liquid-water through the snowpack were developed with data obtained from the Central Sierra Snow Laboratory Lysimeter during April and May of 1954. The equations apply to a "ripe" snowpack (well-aged snow with a spherical crystalline structure). However, they are used under all conditions since there is a lack of data and knowledge on the transmission of water through fresh snow. The excess liquid-water is first lagged and then attenuated. The equation for lag is (shown graphically on Figure 3-4):

$$\mathrm{LAG} = 5.33 \cdot [1.0 - \exp(-0.03 \cdot \mathrm{WE}/\mathrm{EXCESS})], \tag{3.19}$$

where: LAG = lag in hours, and
EXCESS = excess liquid water in inches/six hours.

The equation for attenuation is (shown graphically on Figure 3-5):

$$PACKRO = (S + I_1)/[0.5 \cdot \exp(-83.5 \cdot I_1/WE^{1.3}) + 1.0], \qquad (3.20)$$

where: PACKRO = snowpack outflow in inches/six hours;
S = the amount of excess liquid-water in storage in the snowpack at the beginning of the period - inches, and
$I_1$ = the amount of lagged inflow for the current period - inches/six hours.

The functional forms of Eqs. 3.19 and 3.20 were developed by plotting the experimental data. Final coefficient values were determined by minimizing the squared error between simulated and observed snowpack outflow from the lysimeter.

### 3.3.6 HEAT EXCHANGE AT THE SOIL-SNOW INTERFACE

Heat exchange at the soil-snow interface is usually negligible compared to heat exchange at the air-snow interface. In some watersheds a small amount of melt takes place continuously at the bottom of the snowpack and is enough to sustain base flow throughout the winter. The model assumes that a constant amount of melt takes place at the soil-snow interface. This constant rate of melt is defined by the parameter DAYGM which has units of inches of water equivalent/day.

### 3.3.7 COMPONENTS NOT INCLUDED

Neither snowpack sublimation or interception are explicitly included in the model for the following reasons.

a. To calculate snowpack sublimation with reasonable accuracy, dew point and wind data are needed. In this model, neither of those quantities are known. Snowpack sublimation is usually of the same order of magnitude from one snow season to the next for a given watershed. Thus, to some extent the value of SCF would reflect sublimation losses as well as precipitation gage catch deficiencies.

b. Interception of snow by vegetation and any subsequent loss are complex processes. During a storm, interception storage increases until some maximum is reached. After the storm, some of the intercepted snow falls to the ground, some melts and runs down the tree trunks, and some sublimates. Many studies have represented the seasonal loss by interception as a percentage of the total seasonal snowfall. If this is a valid assumption, which it seems to be, then it would be very difficult to separate interception effects from gage catch deficiency effects.

c. In most watersheds the magnitude of sublimation losses and interception losses are much less than the magnitude of precipitation gage catch deficiencies.

## 3.4 SUMMARY OF MODEL PARAMETERS

Following is a list of the parameters used in the snow accumulation and ablation model and their definitions for use as a reference:

a. PXTEMP — Temperature above which precipitation is assumed to be rain (°F).

b. SCF — Multiplying factor to correct for precipitation gage catch deficiency during periods of snowfall.

c. MBASE — Base temperature for melt computations during non-rain periods (°F).

d. UADJ — Average six-hour wind function during rain on snow events [inches/(in. $H_g$ · 6 hr.)].

e. MFMAX — Maximum non-rain melt factor which occurs on June 21 [inches/(6 hr.·°F)].

f. MFMIN — Minimum non-rain melt factor which occurs on December 21 [inches/(6 hr.·°F)].

g. TIPM — Antecedent temperature index parameter ($0.0 < \text{TIPM} \leq 1.0$).

h. NMF — Maximum value of negative melt factor which occurs June 21 [inches/(6 hr.·°F)].

i. SI — Mean areal water-equivalent above which 100 percent areal snow cover always exists (inches).

j. PLWHC — Percent (decimal) liquid water holding capacity.

k. DAYGM — Daily melt at the soil-snow interface (inches).

l. EFC — Percent (decimal) of area over which evapotranspiration occurs when there is 100 percent snow cover. [Evapotranspiration is modified when snow is on the ground by:

$$EP = EFC \cdot P_e + (1.0 - EFC) \cdot (1.0 - AESC) \cdot P_e, \qquad (3.21)$$

where:

$P_e$ is watershed potential evapotranspiration modified for snow cover (inches), and AESC is percent (decimal) areal extent of snow cover].

Reference:

Linsley, R. K., Kohler, M. A. and Paulhus, J. L. H., Hydrology for Engineers, McGraw-Hill, New York, 1958, 340 pp.

# TABLE 3-1

# SNOW-AIR INTERFACE HEAT EXCHANGE SUMMARY

## A. AIR TEMPERATURE > 32 °F

1. No rain or light rain ( < 0.1"/ 6 hr )

$$\boxed{\text{Heat Exchange} = (T_a - \text{MBASE}) \cdot \text{Melt factor}}$$

2. Rain (≥ 0.1"/ 6 hr )

assume: no solar radiation
longwave equals blackbody radiation at air temperature
dew-point = air temperature
temp. of rain = air temperature

$$\boxed{\begin{gathered}\text{Heat exchange} = 0.007 \cdot (T_a - 32) + \\ 7.5 \cdot \gamma \cdot f(\mu) \cdot (T_a - 32) + 8.5 \cdot f(\mu) \cdot (e_a - 0.18) \\ + 0.007 \cdot \text{Rain} \cdot (T_a - 32)\end{gathered}}$$

$\gamma$ = psychrometric constant, $e_a$ = vapor pressure
$f(\mu)$ = wind function

## B. AIR TEMPERATURE ≤ 32 °F

$$\boxed{\text{Heat Exchange} = (T_{a_2} - ATI_1) \cdot \text{Negative melt factor}}$$

ATI is antecedent temperature index

$ATI_2 = ATI_1 + TIPM \cdot (T_{a_2} - ATI_1)$

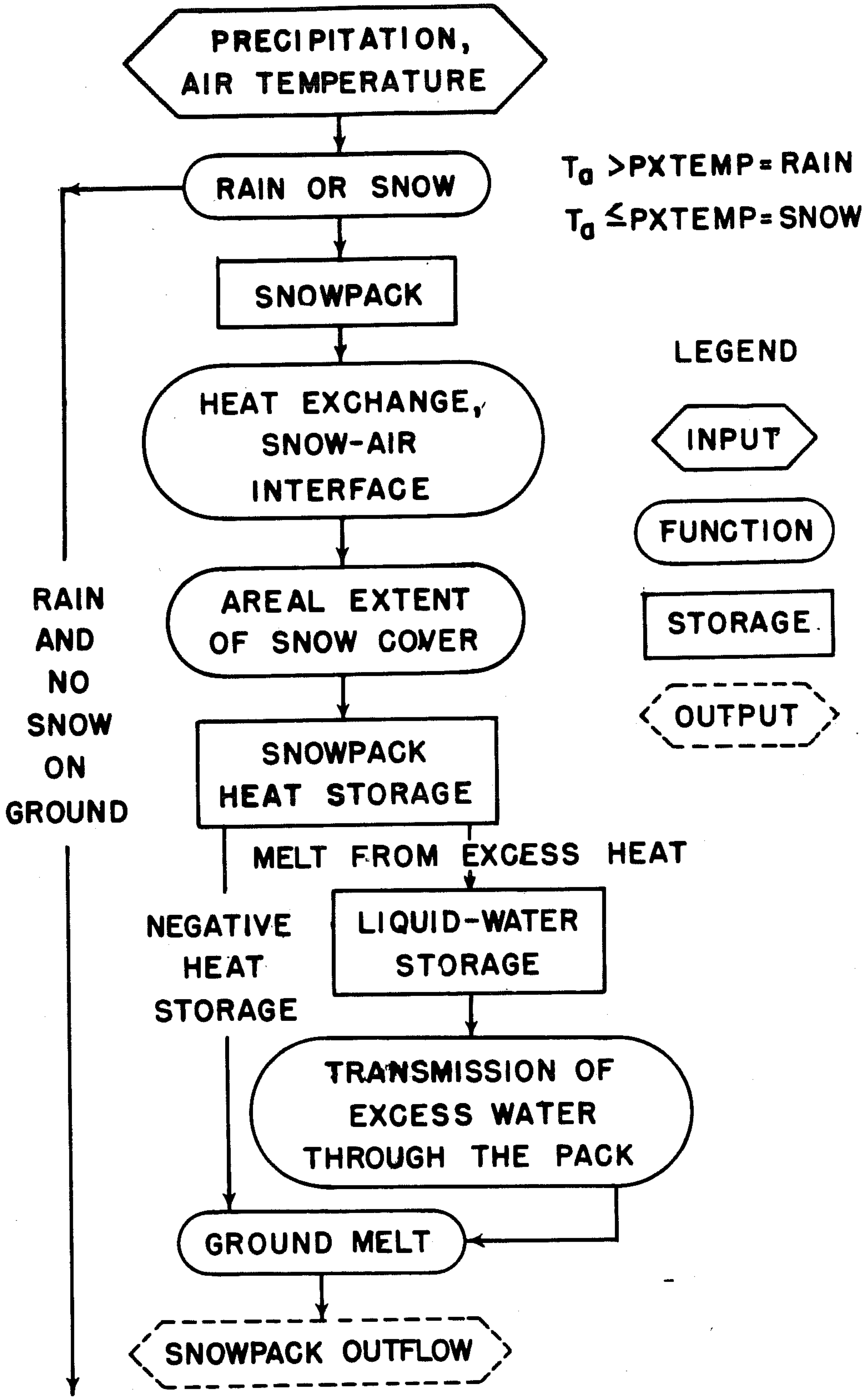


Figure 3-1. - Flow chart of snow accumulation and ablation model.

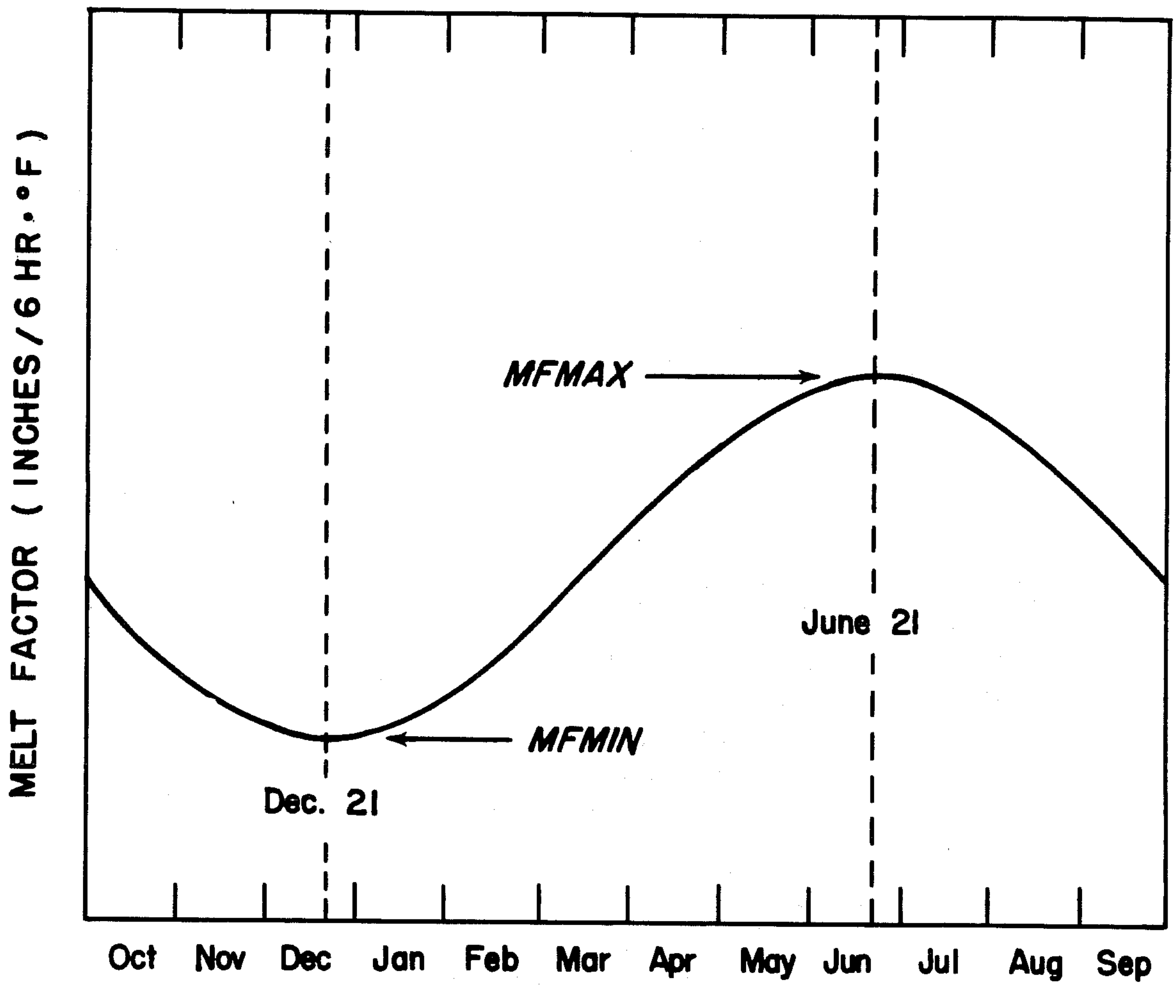


Figure 3-2. - Seasonal variation in melt factors.

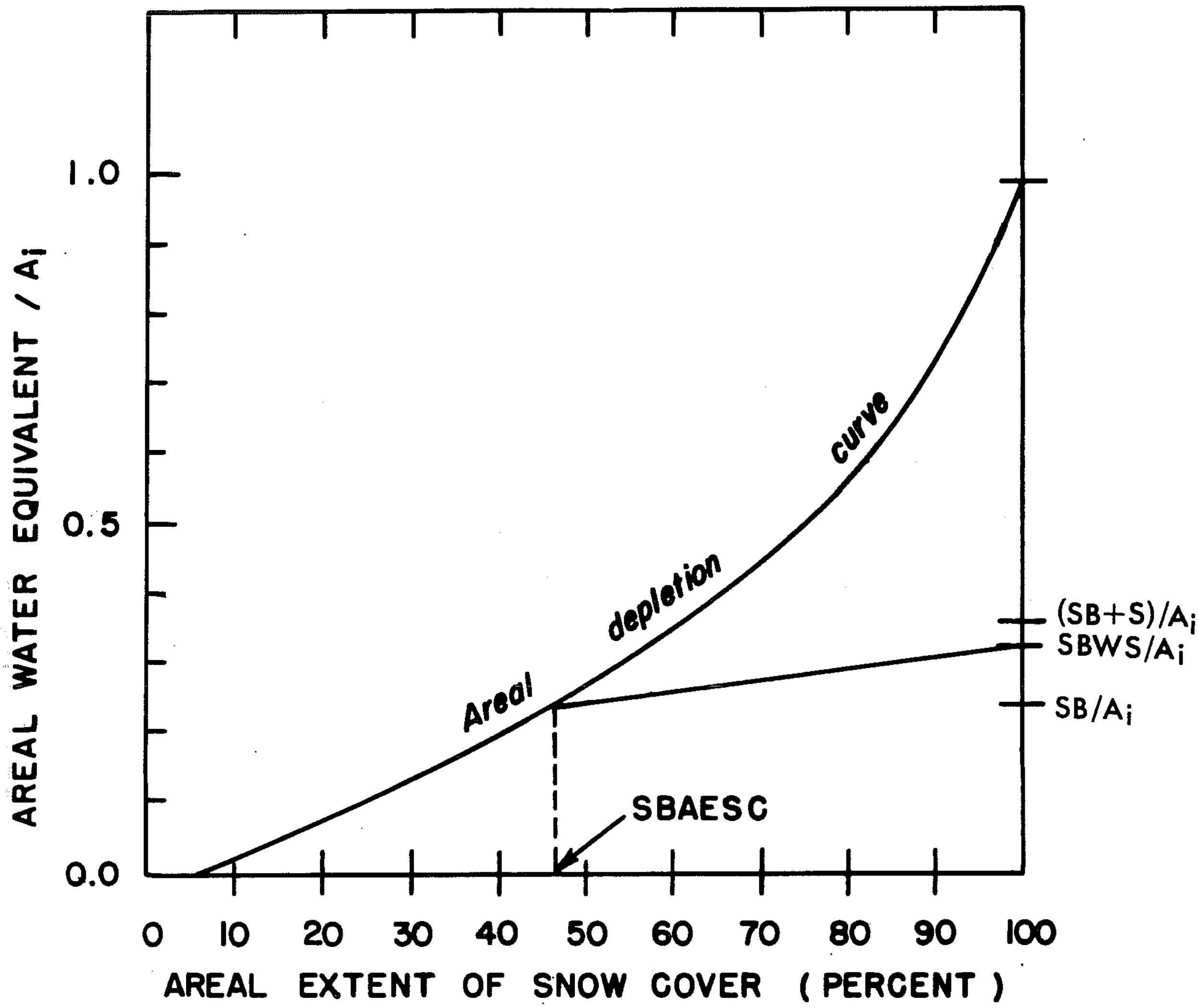


Figure 3-3. - Snow cover areal depletion curve.

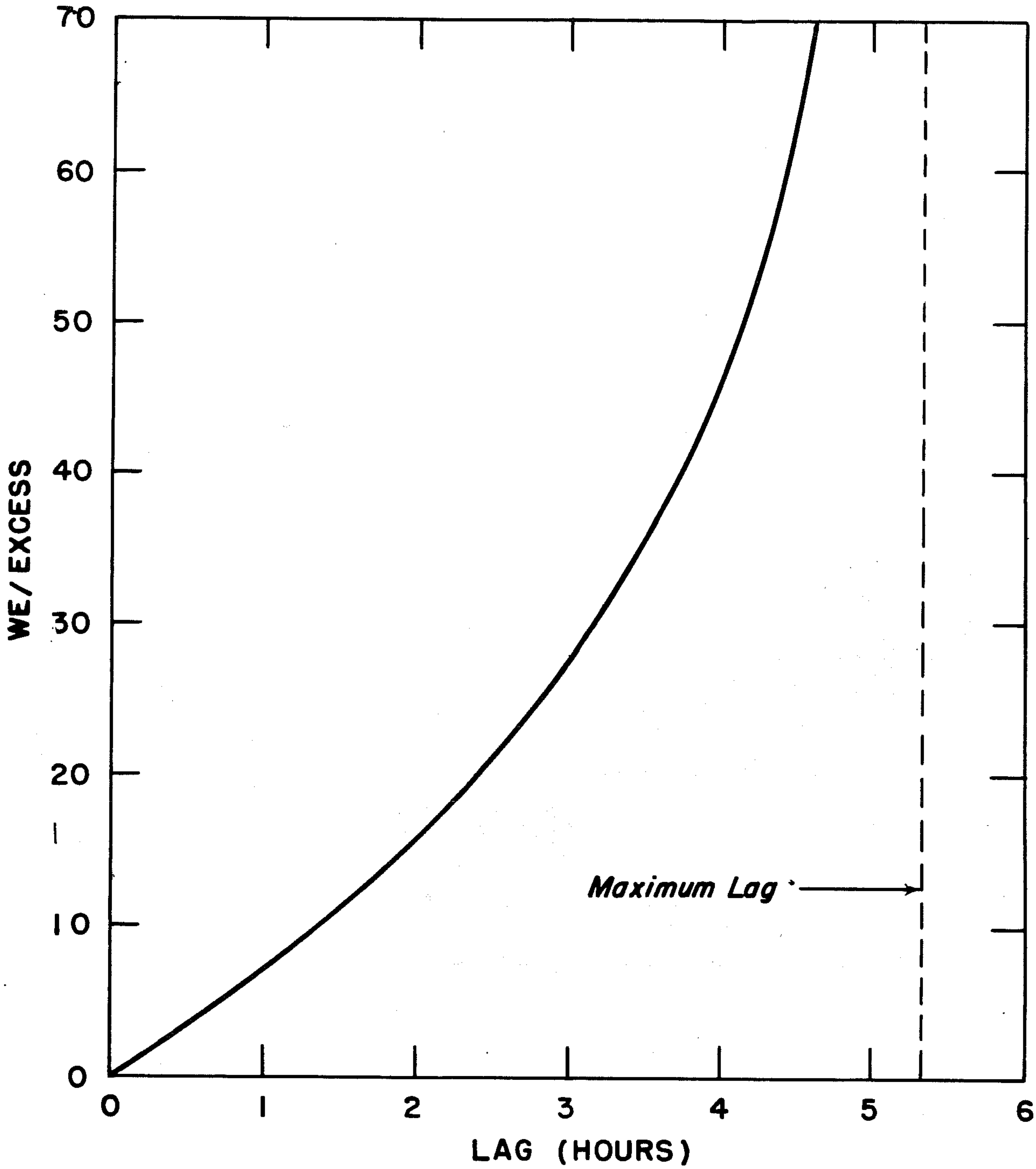


Figure 3-4. - Lag applied to excess liquid-water moving through a snowpack.

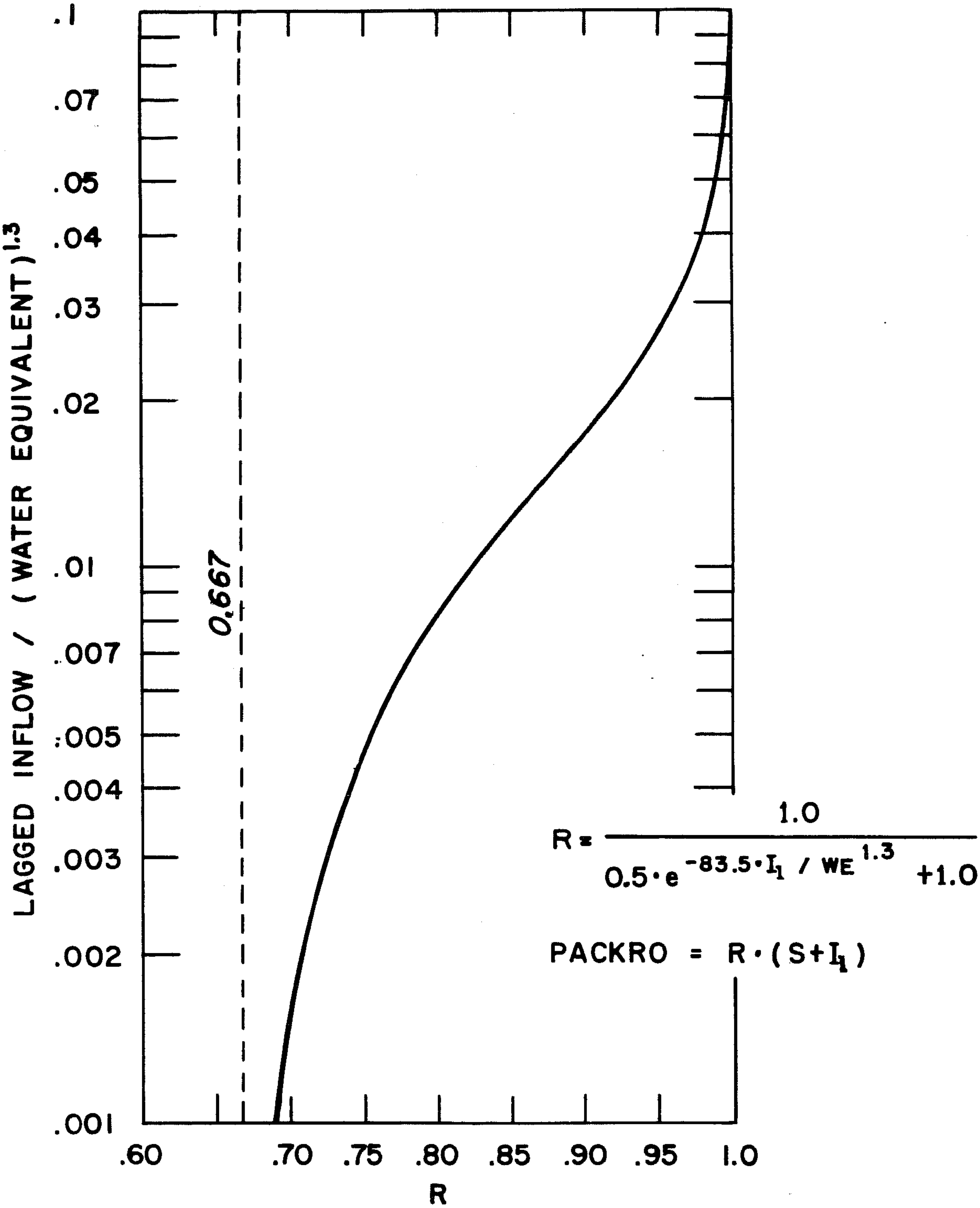


Figure 3-5. - Attenuation of excess liquid water moving through a snowpack.